

Rotation rate?

Viscosity?

Reynolds number?

Boundary
conditions?

Forcing properties?

Aspect ratio?

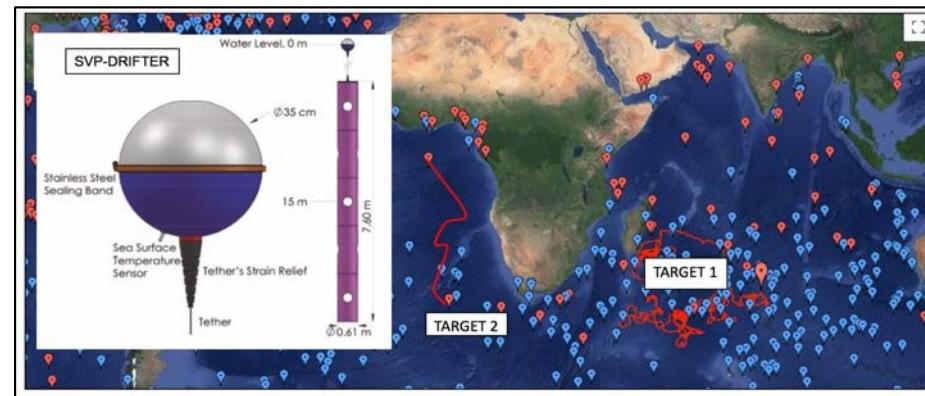
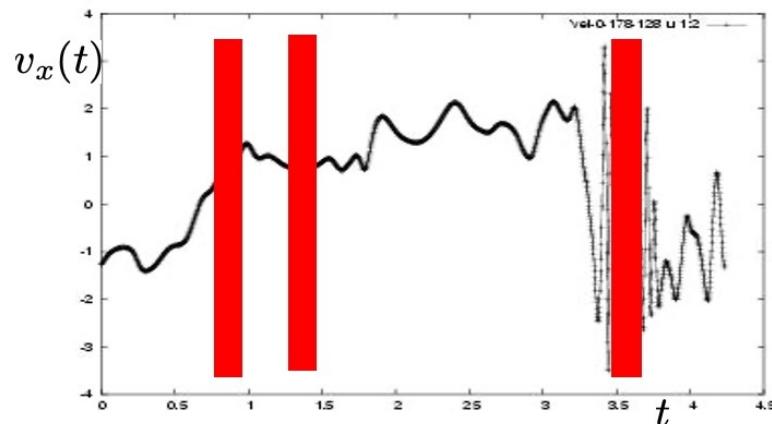
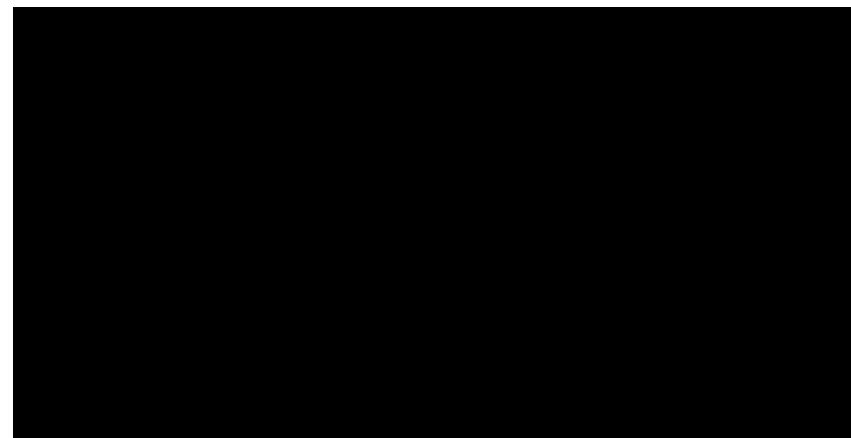
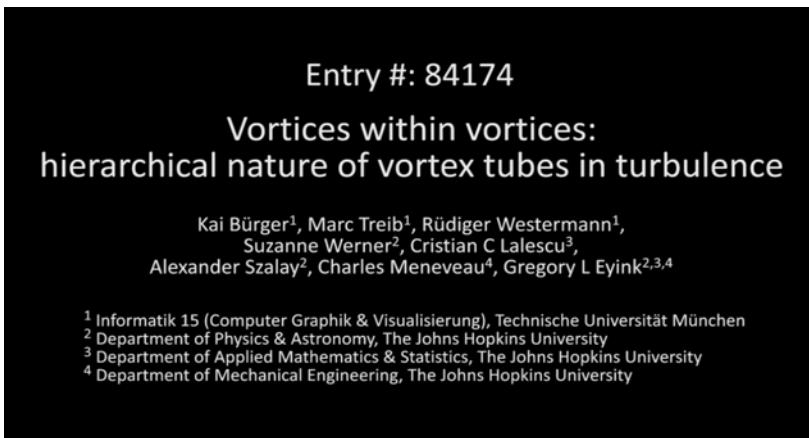


FIG. 1: INSTANTANEOUS WORLDWIDE DISTRIBUTION OF DRIFTERS FROM THE GLOBAL DRIFTER MAP PROGRAM [www0]. WE HIGHLIGHTED IN RED TWO POTENTIAL TARGETS: (1) KEEP THE PROBES INSIDE A GIVEN REGION OR (2) MINIMISING THE NAVIGATION TIME AMONG TWO END-POINTS (ZERMELO PROBLEM).
INSET: A SKETCH OF THE DRIFTER WITH THE LONG DROUZE AT 15M DEPTH.

Machine-learning and equations-informed tools for generation and augmentation of turbulent data.

Artificial Intelligence and the Uncertainty challenge in Fundamental Physics
Paris 2023

1. Short introduction to Eulerian Turbulence
2. Data-driven and Equation-Informed tools for Eulerian Turbulence:
 - a. Linear Principal Orthogonal Decomposition
 - b. Deterministic Generative Adversarial Networks
 - c. Diffusion Models
 - d. Nudging
 - e. Physics Informed NN



TURBULENCE OR TURBULENCES?

MASS X ACCELERATION = INTERNAL FORCES + EXTERNAL FORCES

$$\rho[\partial_t v + v \cdot \partial v] = -\partial p + \nu \Delta v + g\theta + F(B, B) + 2\Omega \times v + \hat{F}_{mech}$$

EULERIAN $\partial_t \theta + v \cdot \partial \theta = \chi \partial^2 \theta \leftarrow$ TEMPERATURE

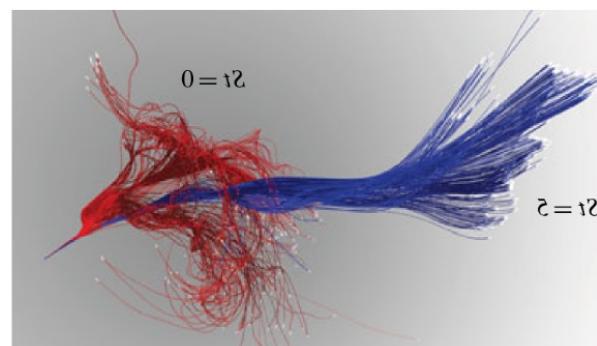
$$\partial_t B + v \cdot \partial B = B \cdot \partial v + \chi \partial^2 B \leftarrow$$
 MAGNETIC FIELD

$$\partial \cdot v = 0$$

+ BOUNDARY CONDITIONS: (2D, 3D, THIN/THICK LAYERS ETC...)

LAGRANGIAN

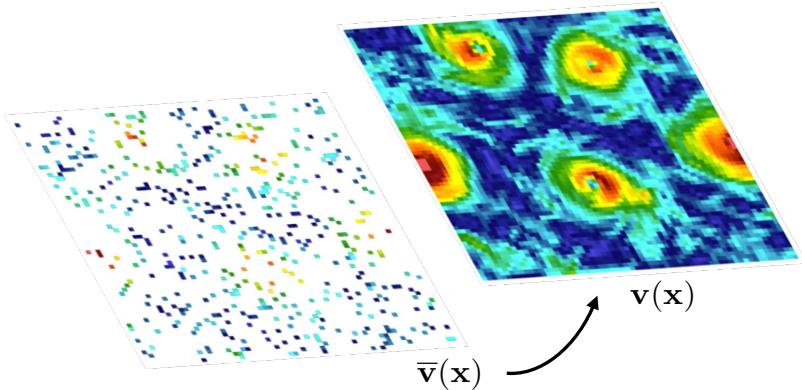
$$\dot{X}(t) = v(X(t), t)$$



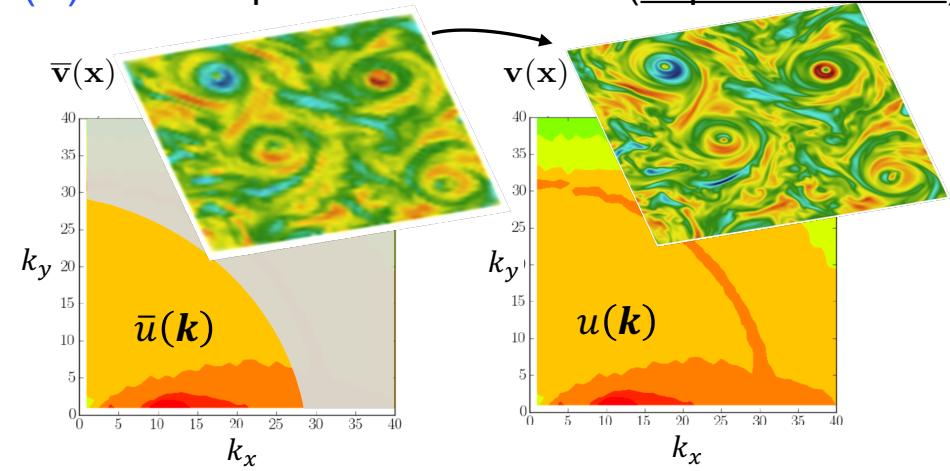
A. Alexakis, L. B. , Cascades and transitions in turbulent flows. Phys. Rep. 767, 1–101 (2018)

RECONSTRUCTION OF MISSING INFORMATION FEATURES RANKING: QUALITY AND QUANTITY OF DATA

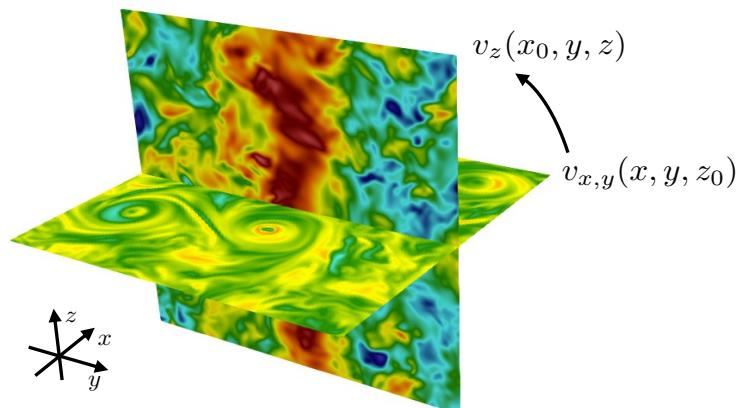
(i) Real-space Reconstruction (full state)



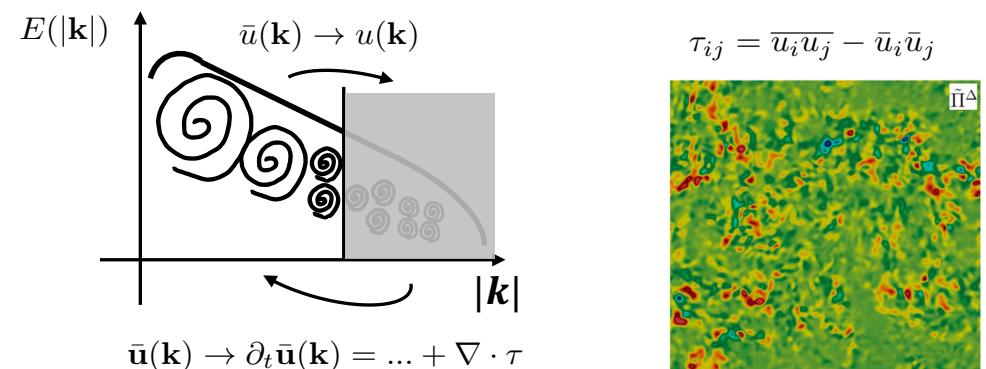
(iii) Fourier-space Reconstruction (Super Resolution)



(ii) Missing Physics (Inverse Problems)



(iv) Sub-Grid Modeling

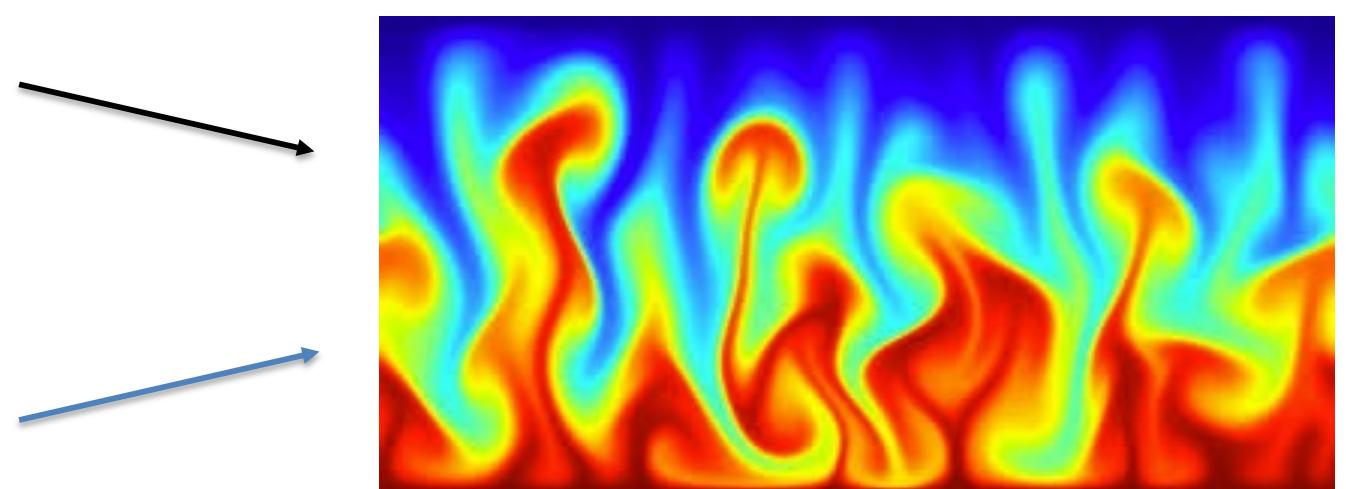
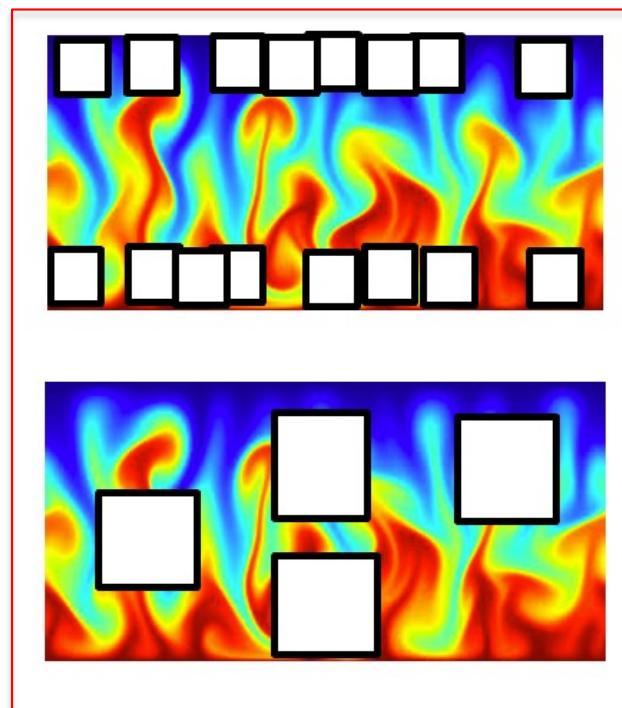


M. Buzzicotti. "Data reconstruction for complex flows using AI: recent progress, obstacles, and perspectives." *Europhysics Letters*, EPL 142 23001 (2023).

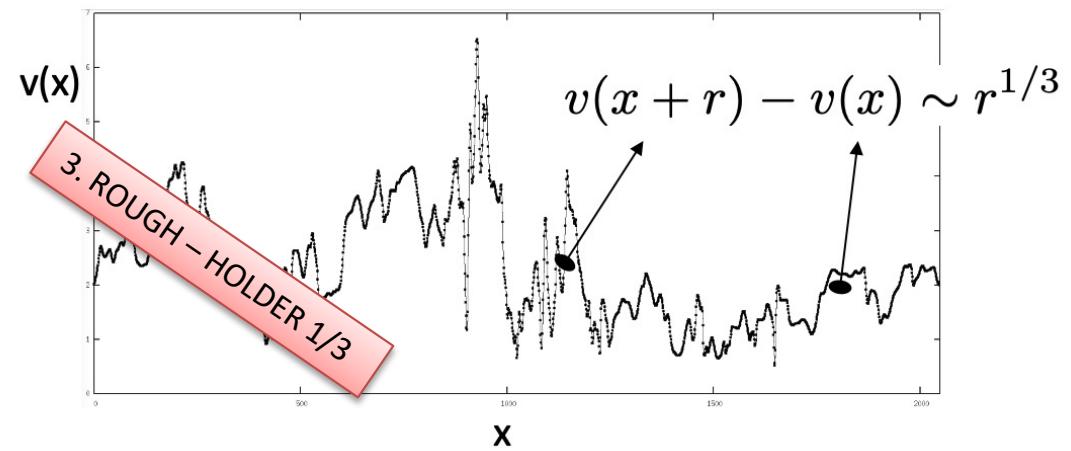
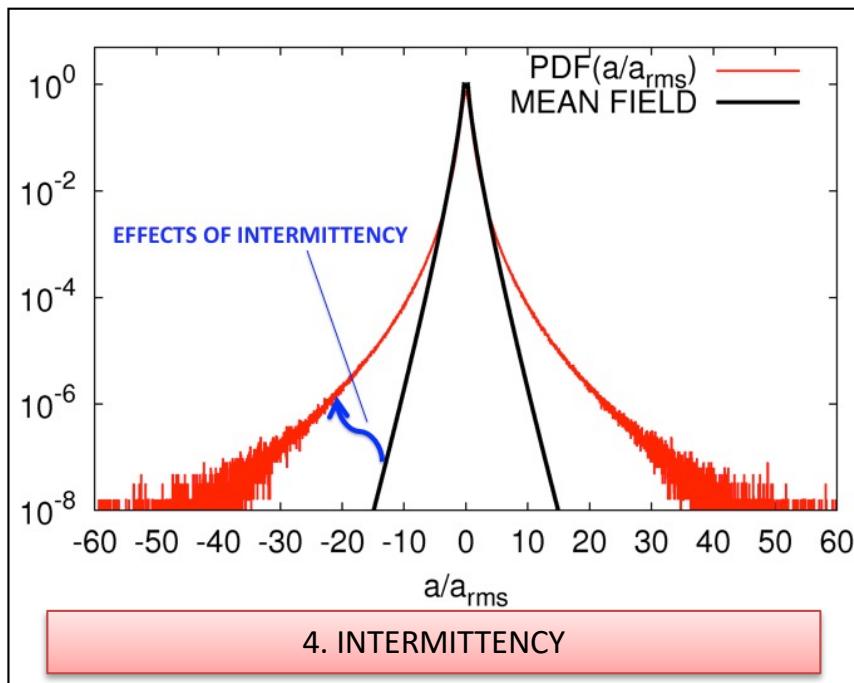
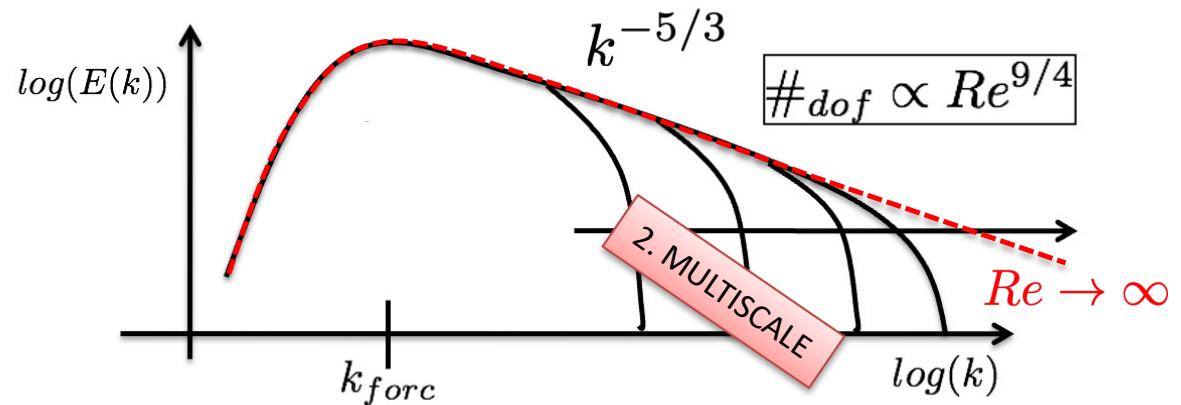
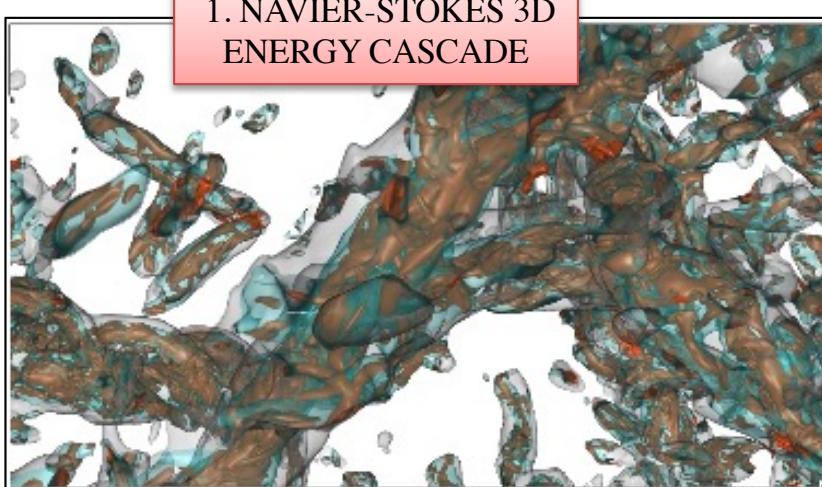
WHY ?

1. GENERATION OF MISSING INFORMATION (INPAINTING/SUPER-RESOLUTION) 2. FEATURES RANKING: WHAT IS THE BEST METRIC TO RECONSTRUCT TURBULENT DATA?

- IT IS MORE DIFFICULT TO RECONSTRUCT SPATIAL OR TEMPORAL DATA?
- HOW MANY DATA/VARIABLES YOU NEED TO SUPPLY FOR PERFECT RECONSTRUCTION (SYNCHRONIZATION-TO-DATA)?
- CAN YOU INFER VELOCITY FIELDS FROM TEMPERATURE SNAPSHOTS AND/OR VICEVERSA?
- IS IT BETTER TO PROVIDE INFORMATION FROM BOUNDARIES OR BULK?
- FROM LARGE OR SMALL SCALES?
- **DO WE NEED (IS IT USEFUL) TO KNOW THE EQUATIONS?**
- **HOW TO COMPARE EQUATIONS-BASED AND EQUATIONS-FREE MODELS?**



WHY IS IT TOUGH?

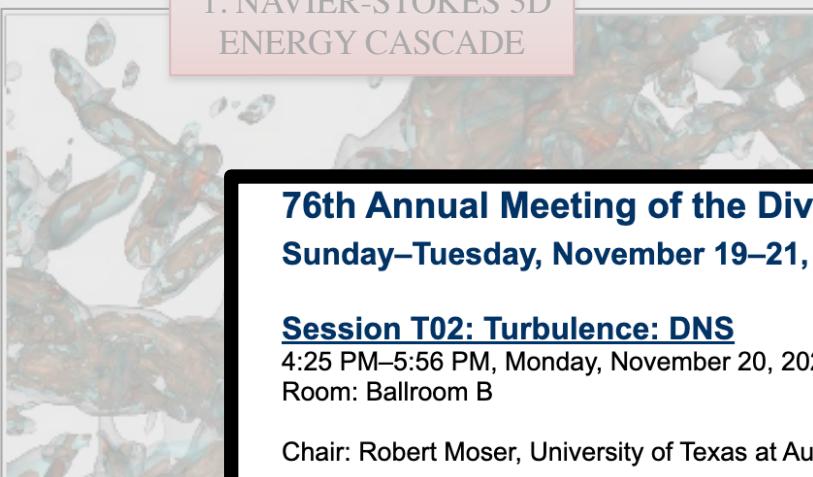


1. OUT-OF-EQUILIBRIUM (NO GIBBS MEASURE)
2. MANY-BODY (INFINITE DEGREES OF FREEDOM)
3. NON-DIFFERENTIABLE FIELDS (HOLDER 1/3)
4. STRONGLY-NON GAUSSIAN

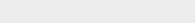
WHY IS IT TOUGH?

1. NAVIER-STOKES 3D ENERGY CASCADE

FIRST EXASCALE COMPUTATION WORLD-RECORD



$\log(E(k))$



76th Annual Meeting of the Division of Fluid Dynamics Sunday–Tuesday, November 19–21, 2023; Washington, DC

Session T02: Turbulence: DNS

4:25 PM–5:56 PM, Monday, November 20, 2023
Room: Ballroom B

Chair: Robert Moser, University of Texas at Austin

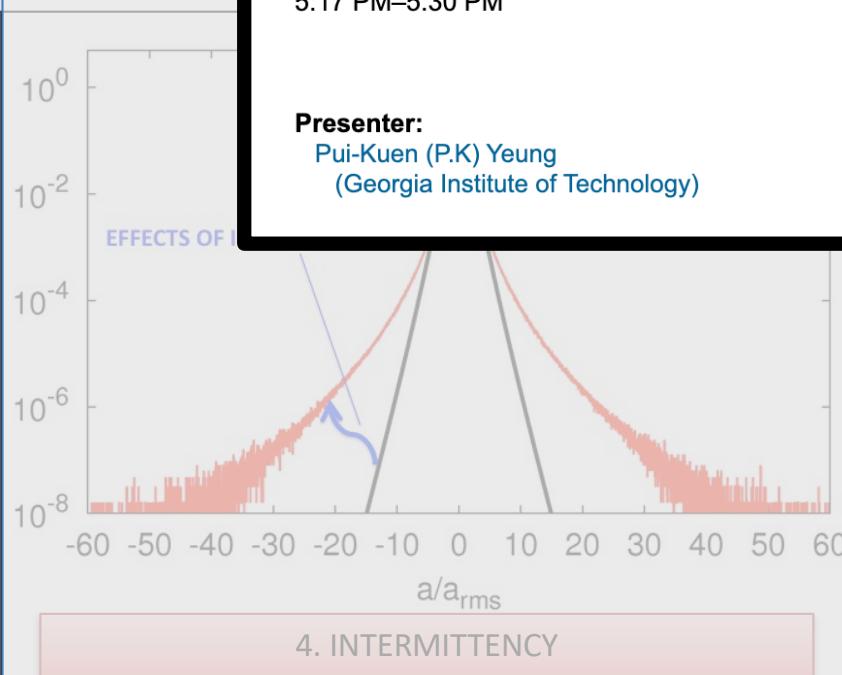
Abstract: T02.00005 : Turbulence simulations at grid resolution up to 32768^3 enabled by Exascale computing*

5:17 PM–5:30 PM

Presenter:

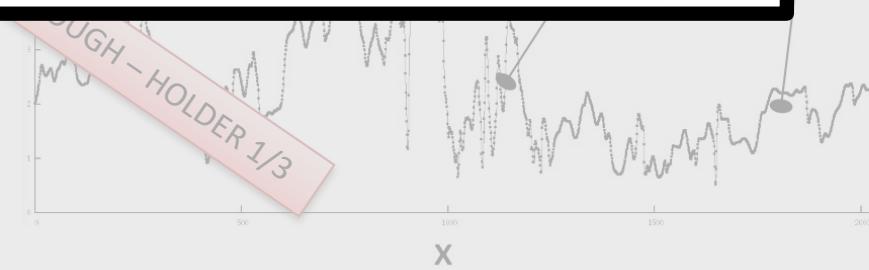
Pui-Kuen (P.K) Yeung
(Georgia Institute of Technology)

$32768^3 \sim 35$ TRILLION GRID POINT
1 CONF ~ 1 PETABYTE
FEATURES RANKING!!!



4. INTERMITTENCY

1. OUT-OF-EQUILIBRIUM (NO GIBBS MEASURE)
2. MANY-BODY (INFINITE DEGREES OF FREEDOM)
3. NON-DIFFERENTIABLE FIELDS (HOLDER 1/3)
4. STRONGLY-NON GAUSSIAN



$\propto Re^{9/4}$

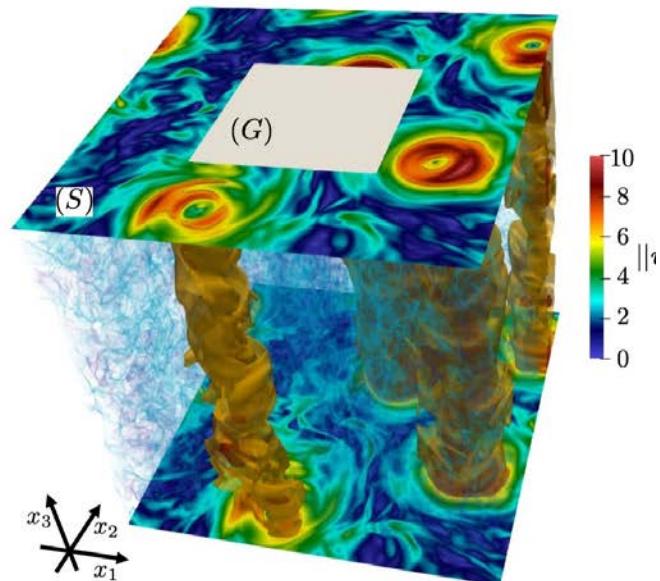
$Re \rightarrow \infty$

$E(k)$

$x) \sim r^{1/3}$

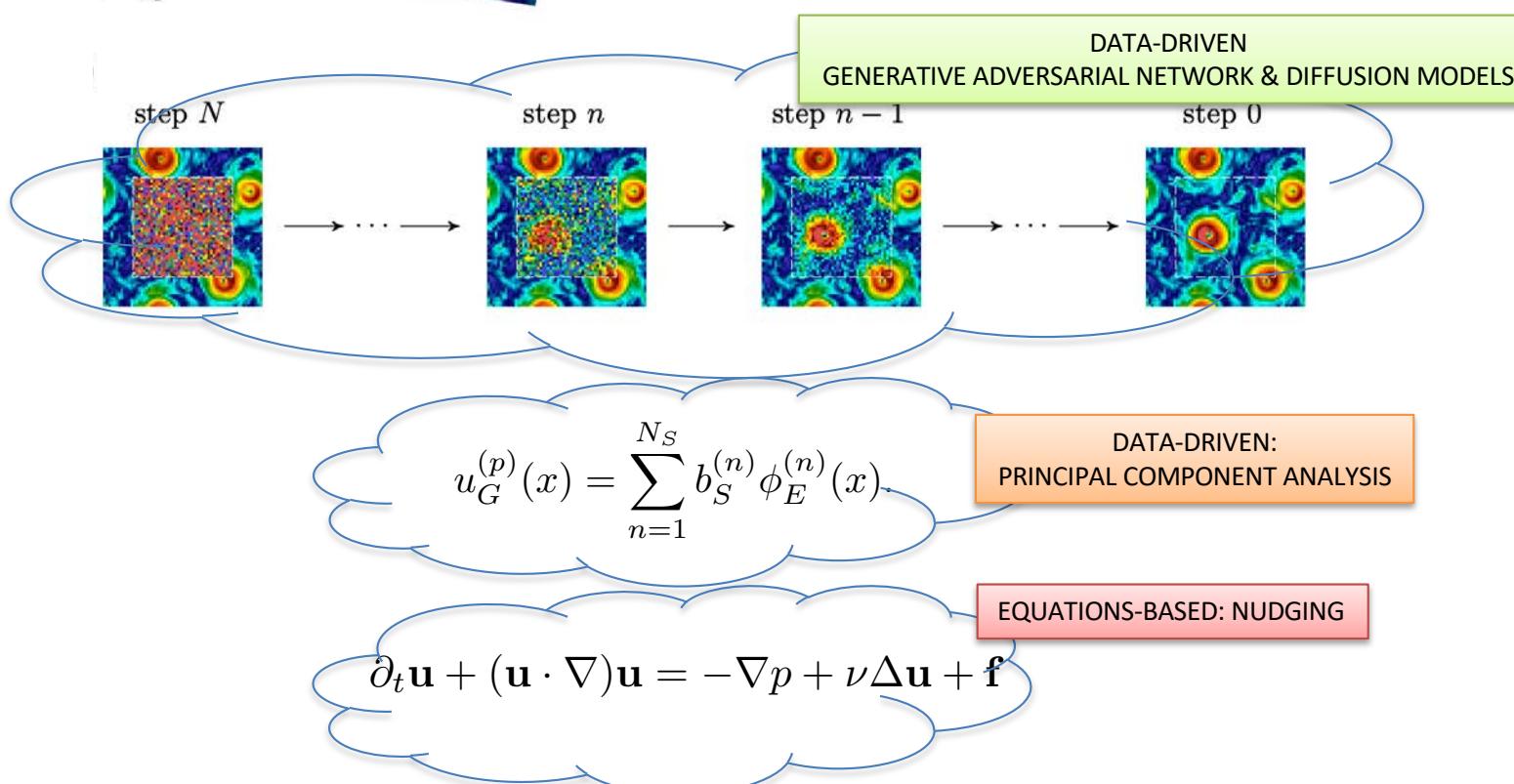
grid resolution: 1024^3

Coherent Structures and Extreme Events in Rotating Multiphase Turbulent Flows.
LB, F. Bonaccorso, I. M. Mazzitelli, et al. Phys. Rev. X **6**, 041036 – 2016

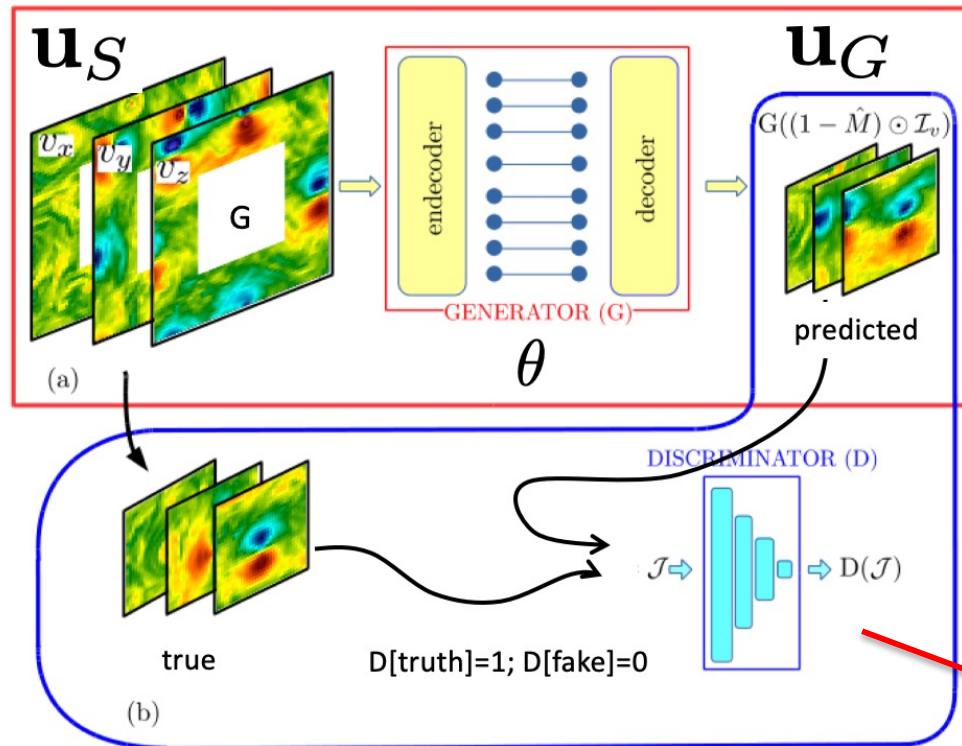


3D TURBULENCE UNDER ROTATION

MOCK SATELLITE MEASUREMENTS



NONLINEAR GENERATIVE ADVERSARIAL NETWORK: CONTEXT ENCODER (ACTOR-CRITIC)



MINIMIZE:

$$\mathcal{L}_{GEN} = (1 - \lambda_{adv}) \mathcal{L}_{MSE} + \lambda_{adv} \mathcal{L}_{adv},$$

$$\begin{aligned} \mathcal{L}_{MSE} &= \left\langle \frac{1}{A(I)} \int_I \| \mathbf{u}_G^{(p)}(\mathbf{x}) - \mathbf{u}_G^{(t)}(\mathbf{x}) \|^2 d\mathbf{x} \right\rangle \\ \mathcal{L}_{adv} &= \langle \log(1 - D(\mathbf{u}_G^{(p)})) \rangle \\ &= \int p(\mathbf{u}_S) \log[1 - D(GEN(\mathbf{u}_S))] d\mathbf{u}_S \end{aligned}$$

MAXIMIZE:

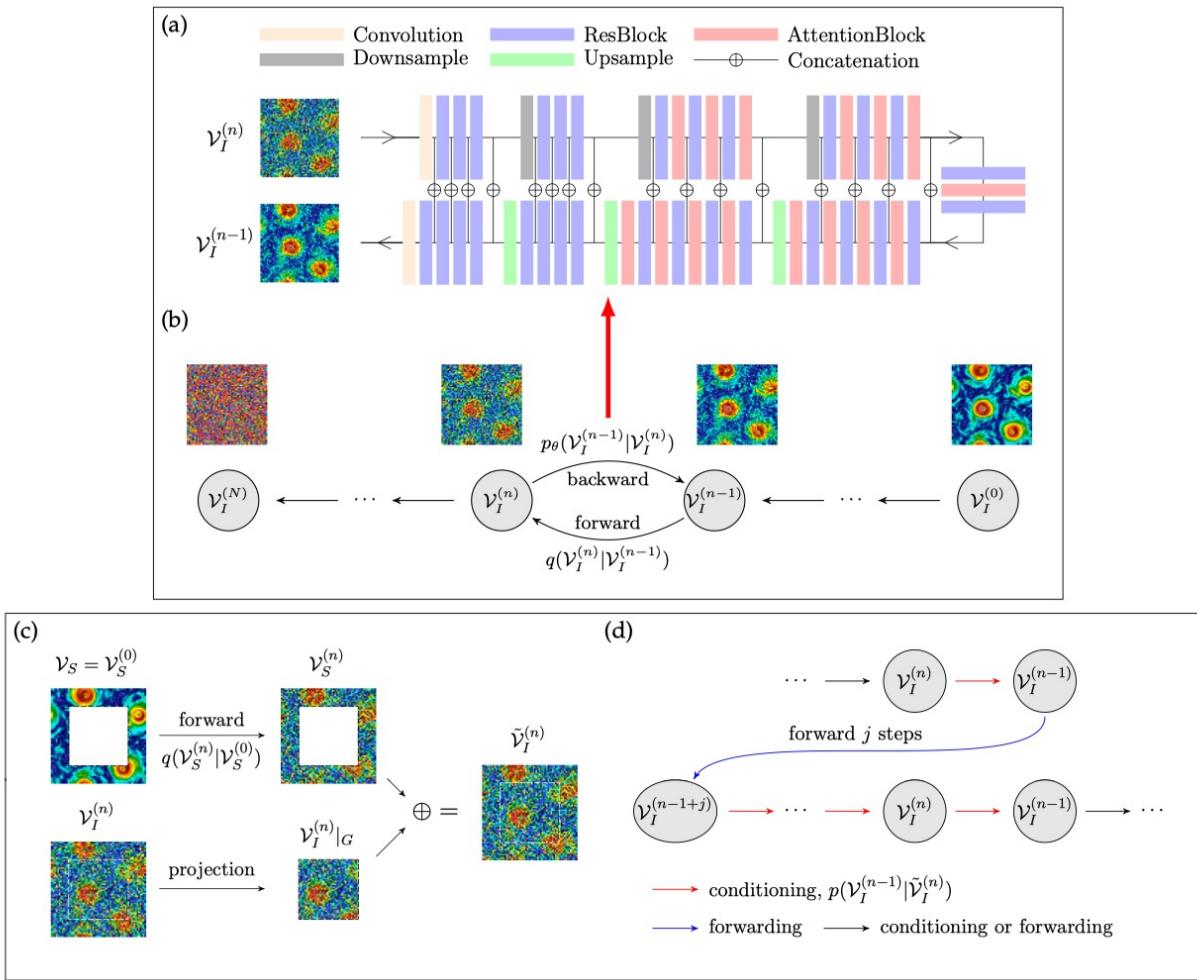
$$\begin{aligned} \mathcal{L}_{DIS} &= \langle \log(D(\mathbf{u}_G^{(t)})) \rangle + \langle \log(1 - D(\mathbf{u}_G^{(p)})) \rangle \\ &= \int [p_t(\mathbf{u}_G) \log(D(\mathbf{u}_G)) + p_p(\mathbf{u}_G) \log(1 - D(\mathbf{u}_G))] d\mathbf{u}_G \end{aligned}$$

$$KL(p_t \parallel p_p) = \int_{-\infty}^{\infty} p_t(x) \log \left(\frac{p_t(x)}{p_p(x)} \right) dx$$

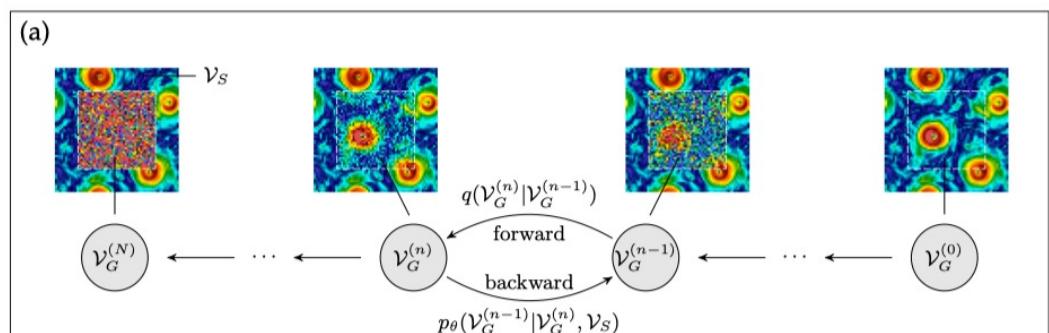
KULLBACK-LEIBLER DISTANCE

1. DIFFUSION MODELS

UNCONDITIONAL GENERATION OUT OF I.I.D. NOISE



RePAINT: UNCONDITIONAL INPAINTING OUT OF I.I.D. NOISE



PALETTE: CONDITIONAL INPAINTING OUT OF I.I.D. NOISE

LINEAR: EXTENDED-PRINCIPAL ORTHOGONAL DECOMPOSITION (EPOD)

$$R_S(x, y) = \langle u(x)u(y) \rangle$$

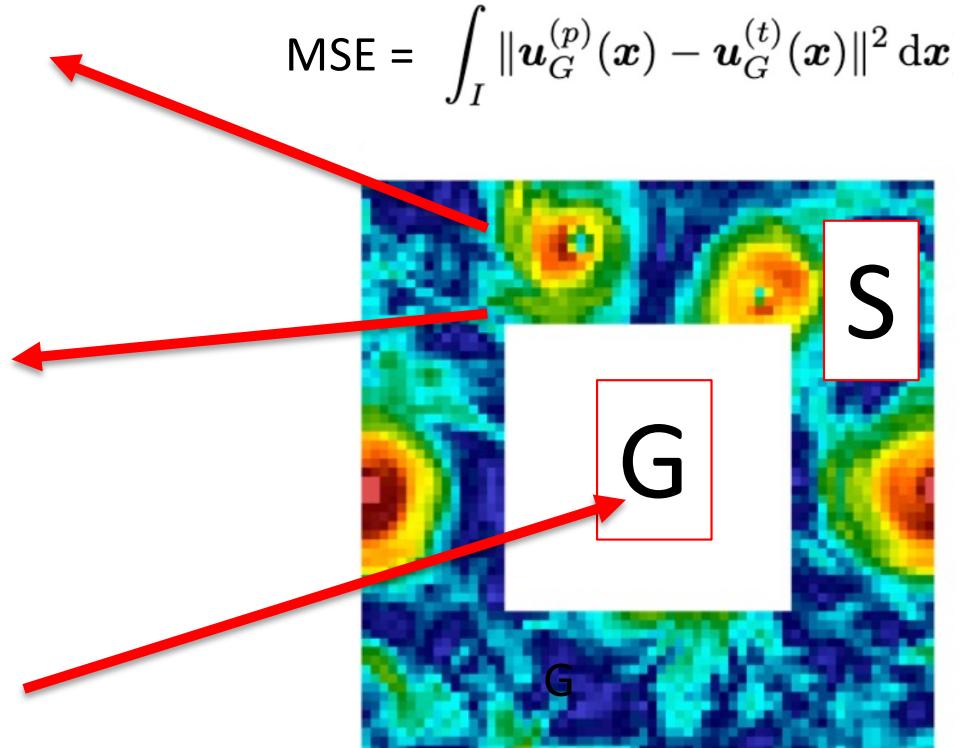
$$\int_{\Omega} R_S(x, y)\phi_S^{(n)}(y) dy = \sigma_n \phi_S^{(n)}(x),$$

$$u_S(x) = \sum_{n=1}^{N_S} b_S^{(n)} \phi_S^{(n)}(x),$$

$$\phi_S^{(n)}(x) = \langle b_S^{(n)} u_S(x) \rangle / \sigma_n.$$

TRAINING

$$\phi_E^{(n)}(x) = \langle b_S^{(n)} u_G(x) \rangle / \sigma_n.$$



RECONSTRUCTION

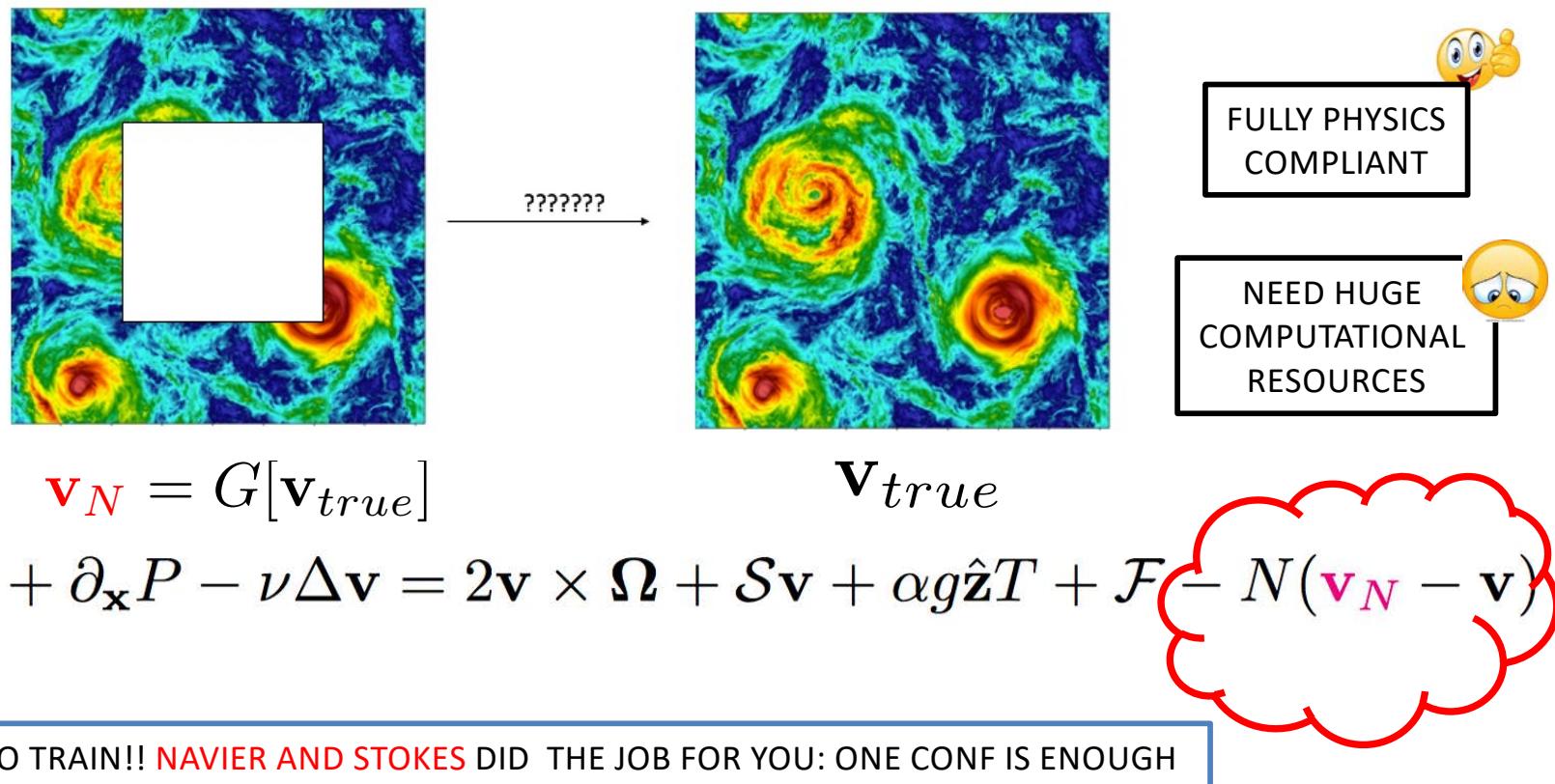
$$u_G^{(p)}(x) = \sum_{n=1}^{N_S} b_S^{(n)} \phi_E^{(n)}(x).$$

J. Boree, Extended POD a tool to analyse correlated events in turbulence. Experiments in Fluids 35(2):188 (2003)

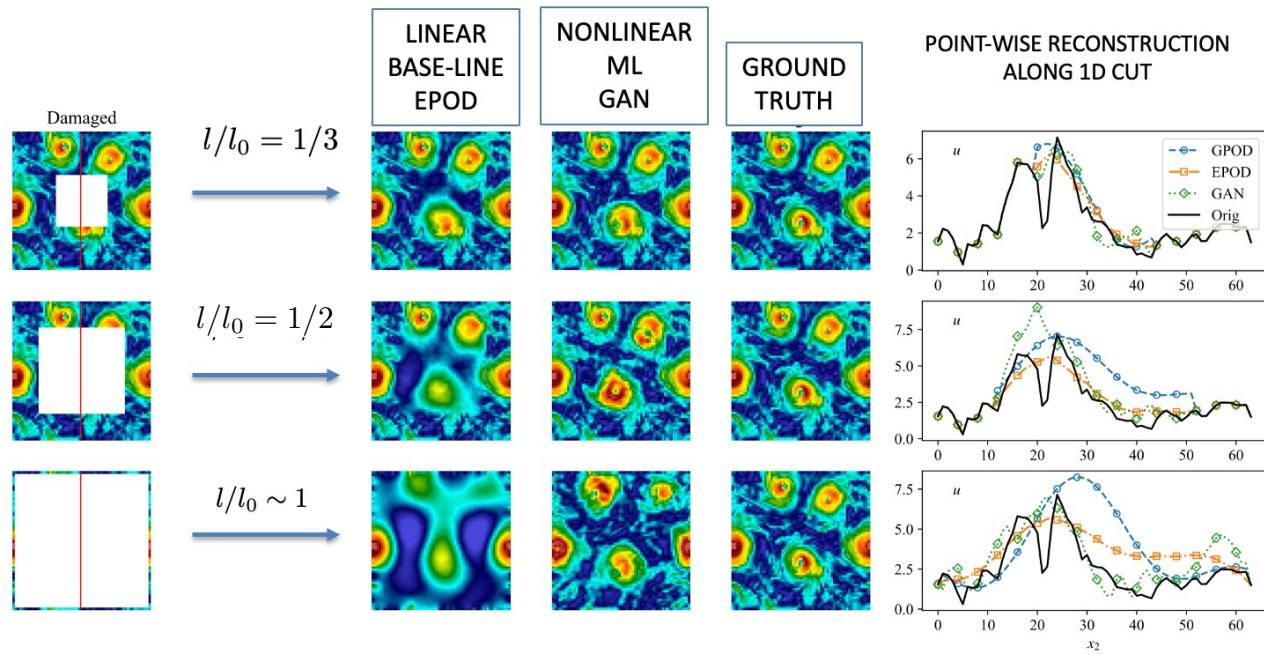
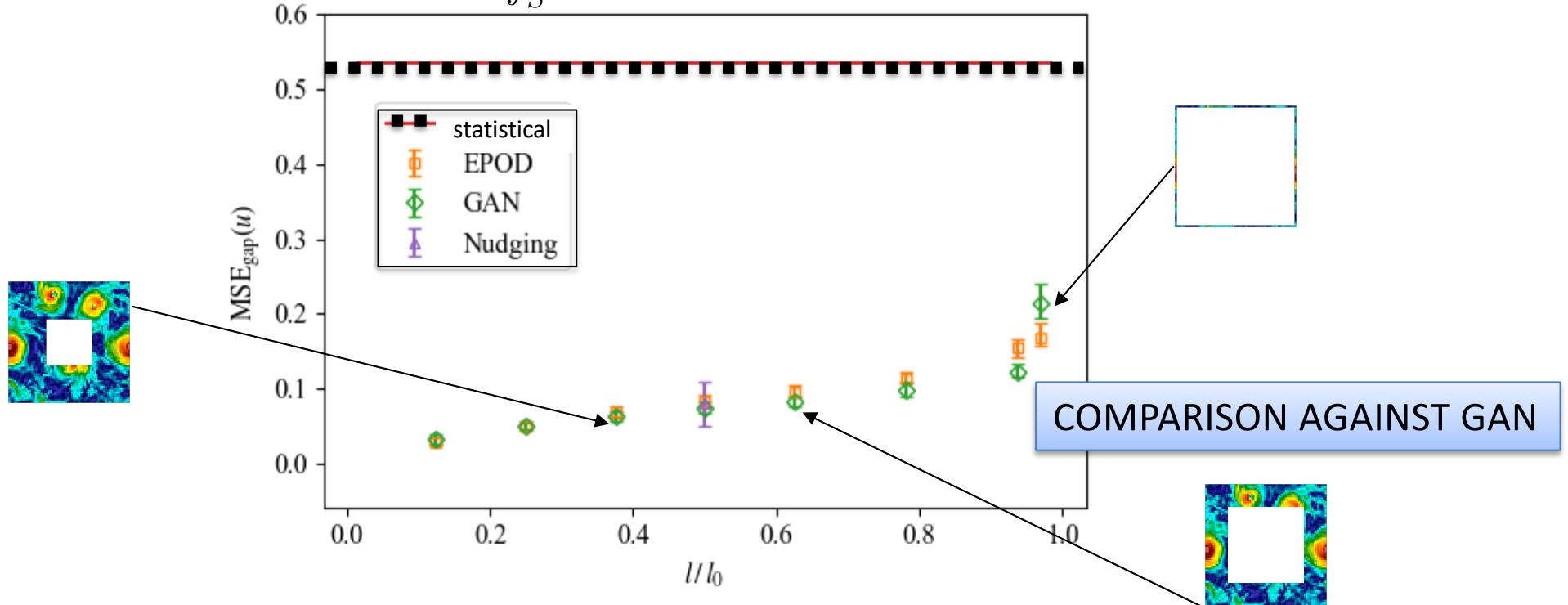
T. Li, L. Buzzicotti, F. Bonacorso, L.B., S. Chen, M. Wan. Data reconstruction of turbulent flows with Gappy-POD and Generative Adversarial Networks. JFM (2023)

NUDGING: AN EQUATION-INFORMED UNBIASED TOOL TO ASSIMILATE AND RECONSTRUCT TURBULENCE DATA/PHYSICS BY ADDING A DRAG TERM AGAINST PARTIAL FIELD MEASUREMENTS

C.C. Lalescu, C. Meneveau and G.L. Eyink. Synchronization of Chaos in Fully Developed Turbulence. Phys. Rev. Lett. 110, 084102 (2013)
 A. Farhat, E. Lunasin, and E.S. Titi. Abridged Continuous Data Assimilation for the 2d Navier-Stokes Equations Utilizing Measurements of Only One Component of the Velocity Field. J. Math. Fluid Mech. 18(1), 1 (2016)



$$\text{MSE} = \langle \int_S d\mathbf{x} (u_{true}(\mathbf{x}) - u_{pred}(\mathbf{x}, \theta))^2 \rangle$$



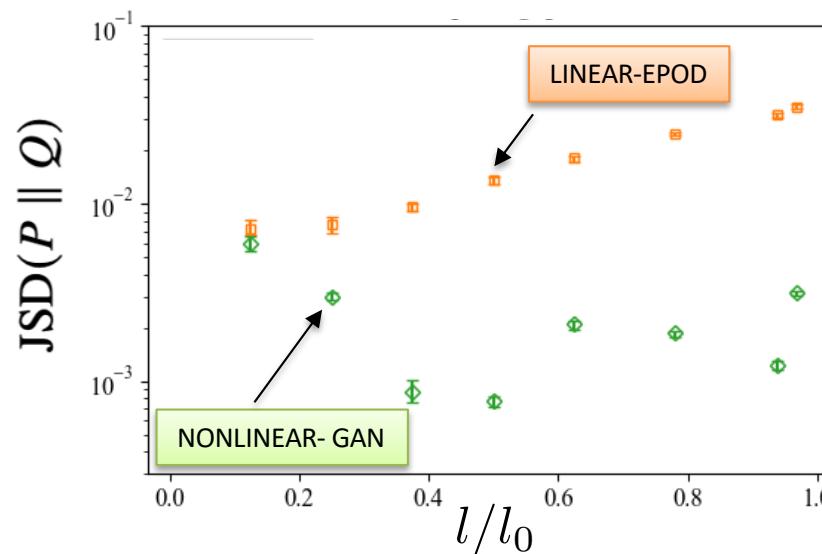
KULLBACK-LEIBLER

$$D(P \parallel Q) = \int_{-\infty}^{\infty} P(x) \log \left(\frac{P(x)}{Q(x)} \right) dx$$

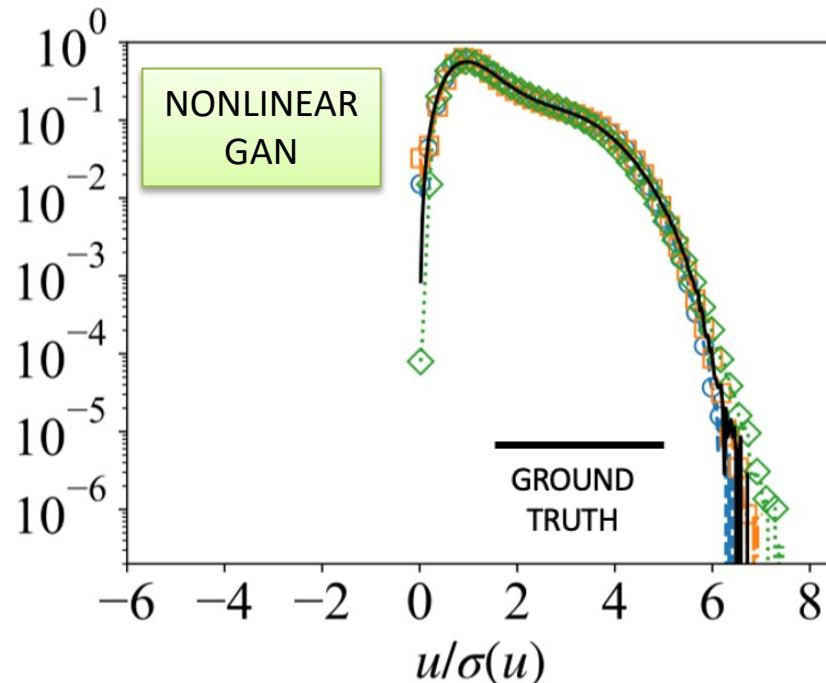
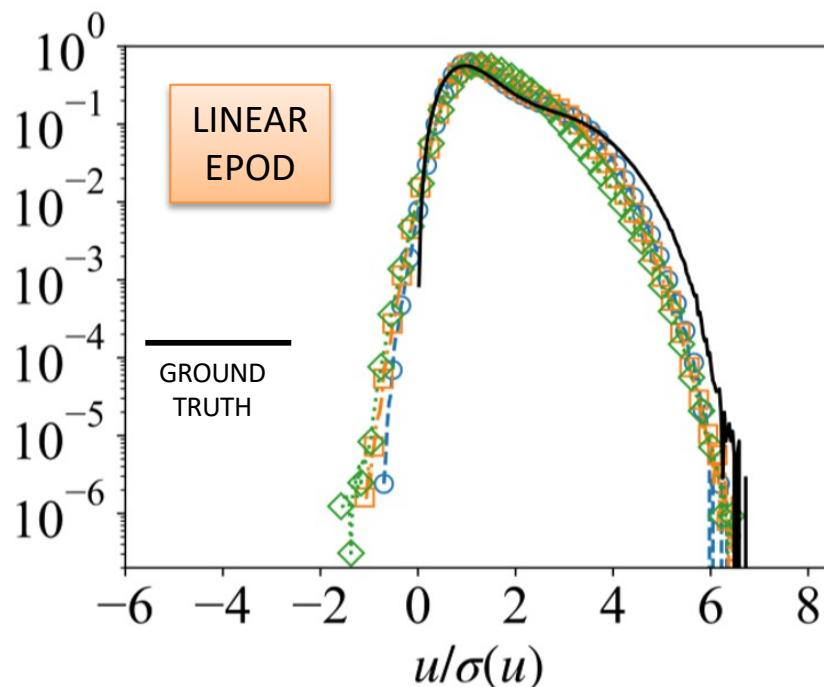
$M = \frac{1}{2}(P + Q)$

JENSEN-SHANNON

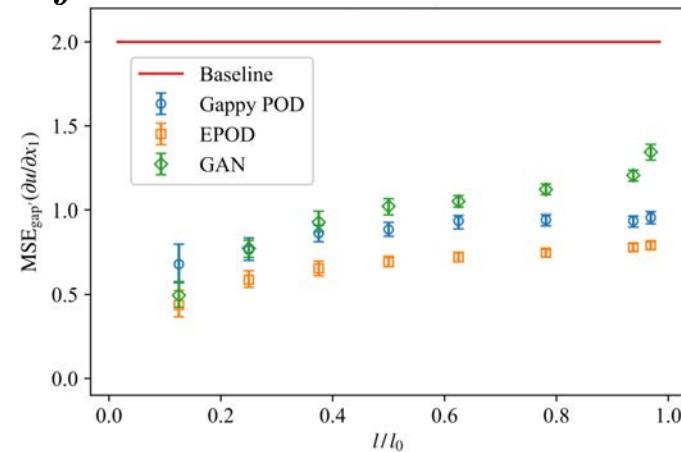
$$\text{JSD}(P \parallel Q) = \frac{1}{2}D(P \parallel M) + \frac{1}{2}D(Q \parallel M),$$



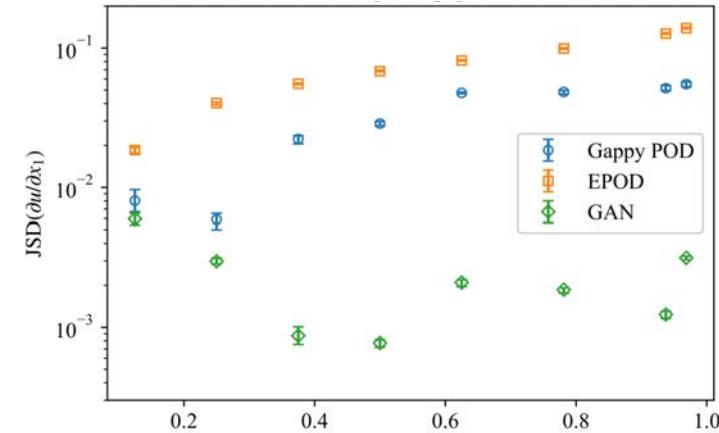
COMPARISON AGAINST GAN



$$\int d\mathbf{x} [\partial_x u^p(\mathbf{x}, \theta) - \partial_x u^t(\mathbf{x})]^2$$



$$JSD(P \parallel Q) = \frac{1}{2} D(P \parallel M) + \frac{1}{2} D(Q \parallel M),$$

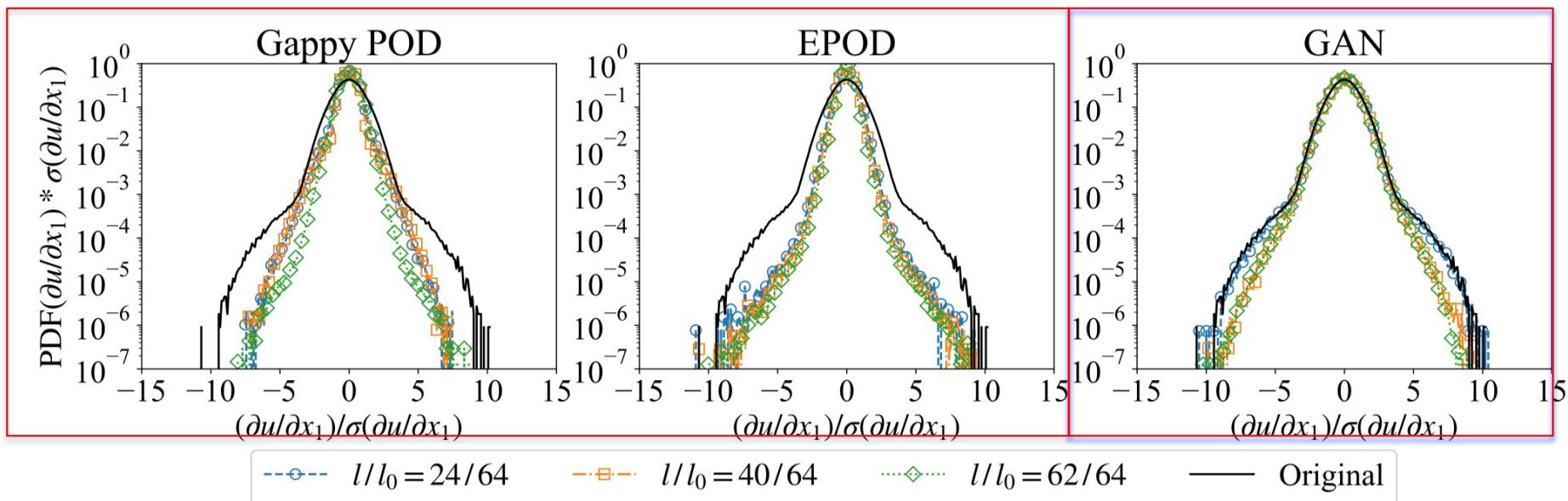


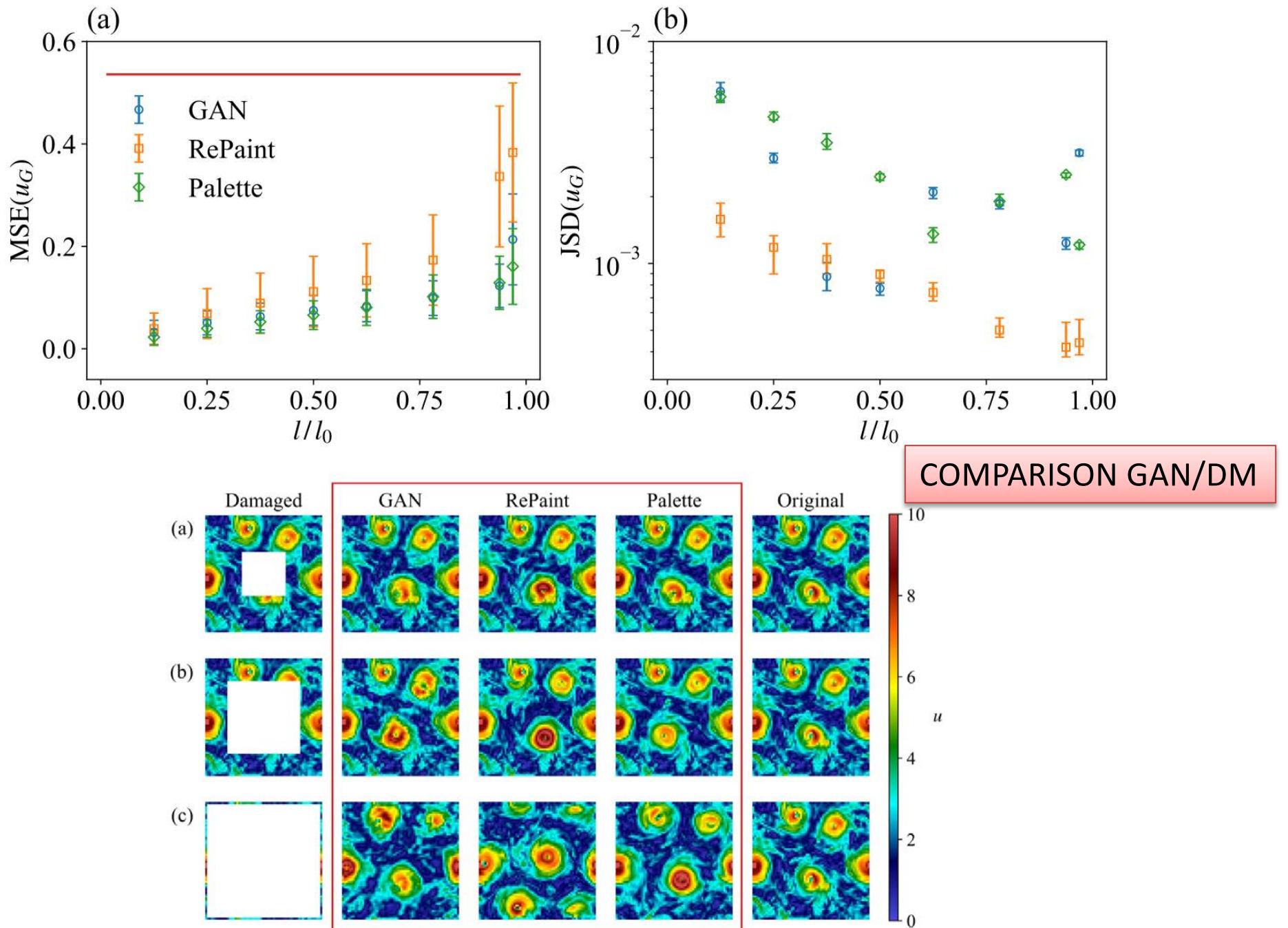
COMPARISON AGAINST GAN

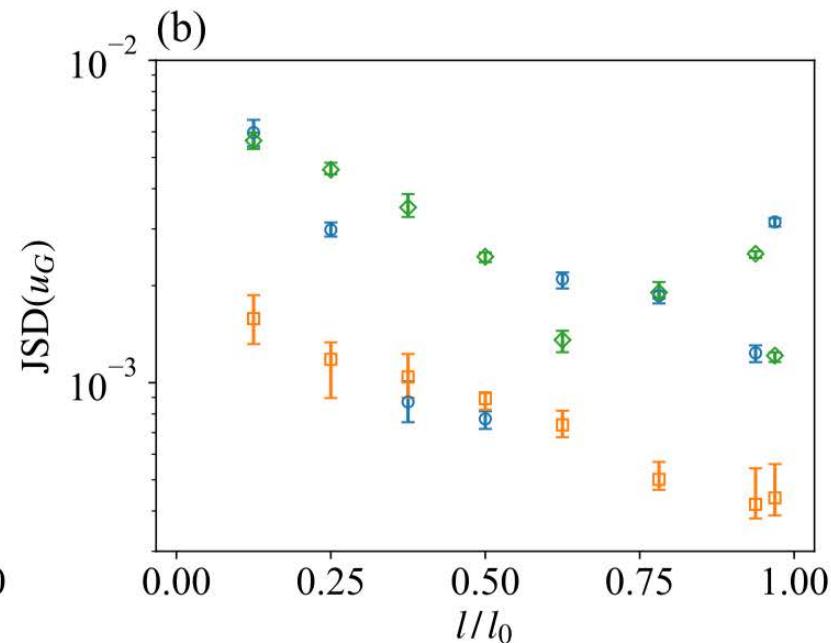
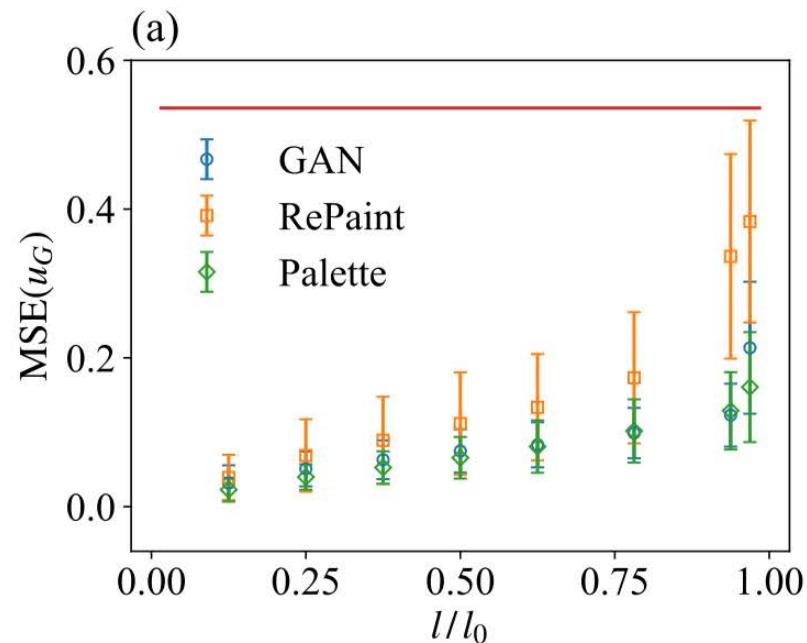
LINEAR GAPPY-POD & EPOD

NONLINEAR GAN

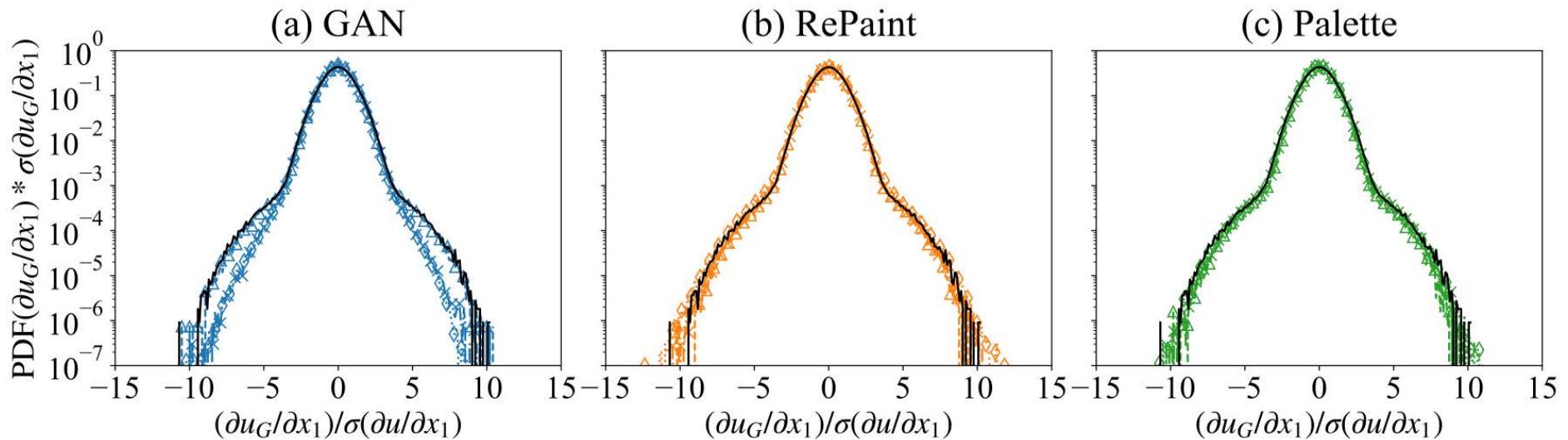
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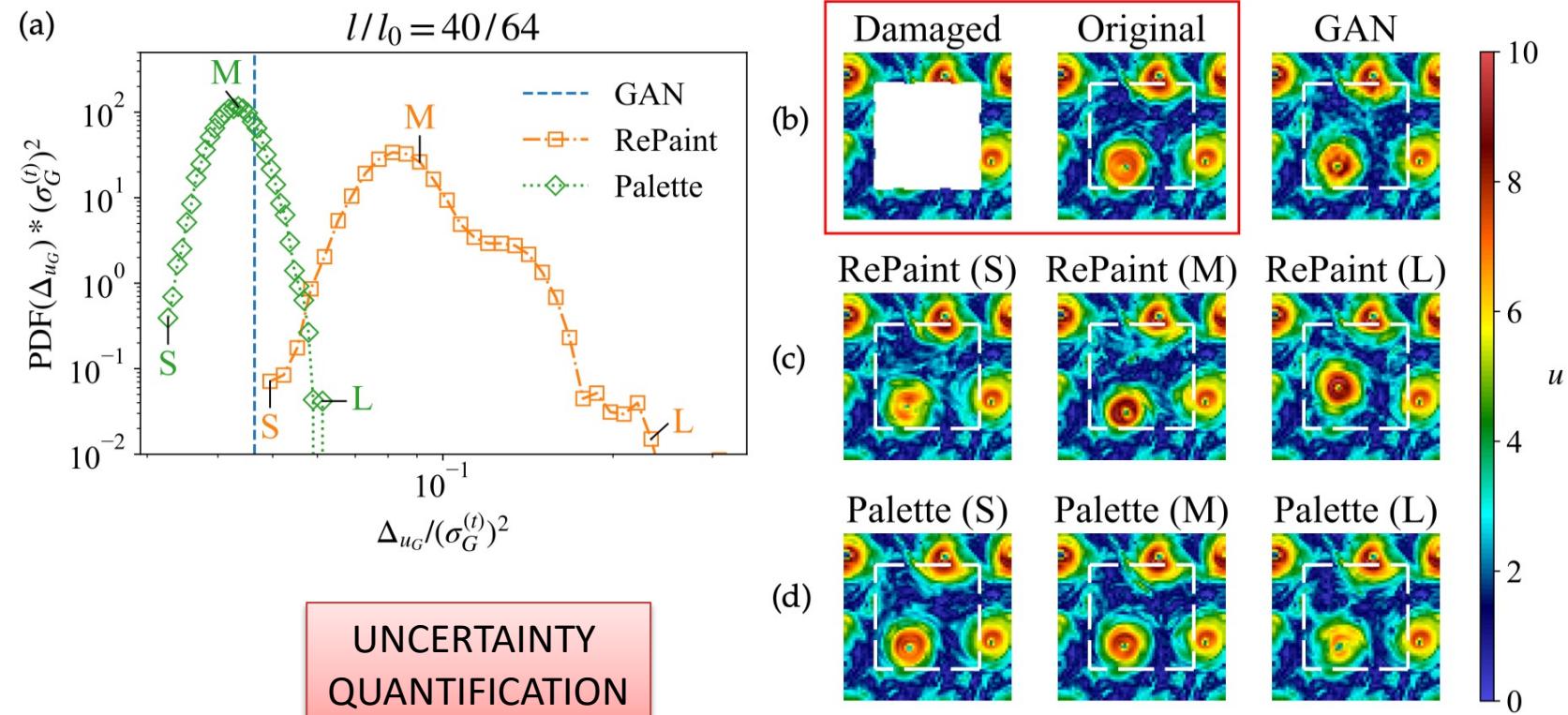
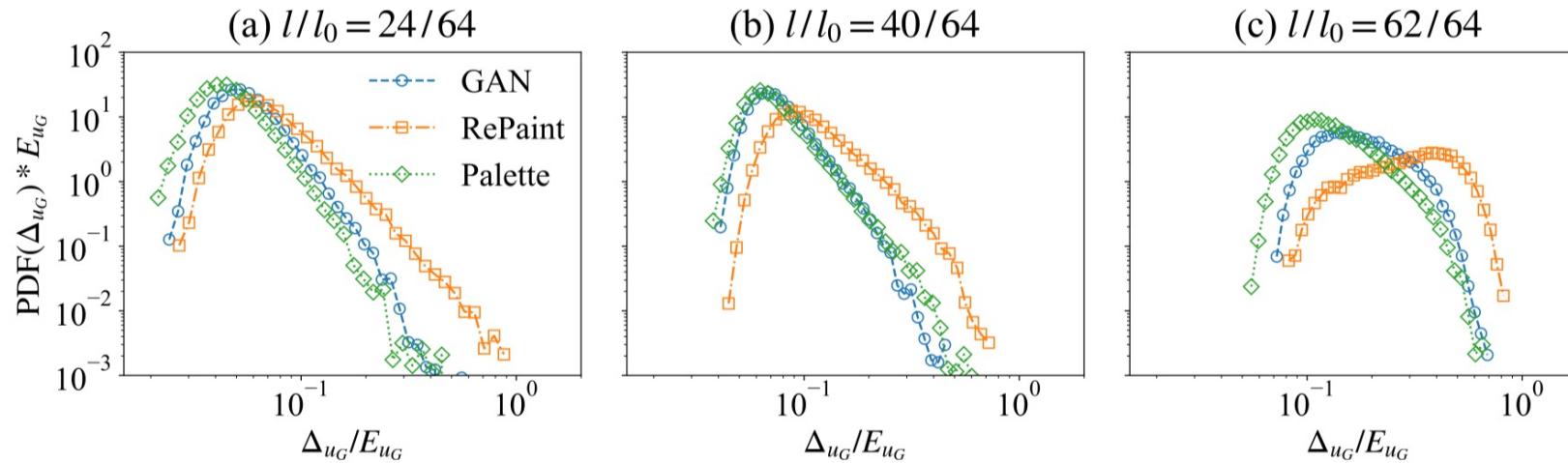


COMPARISON GAN/DM

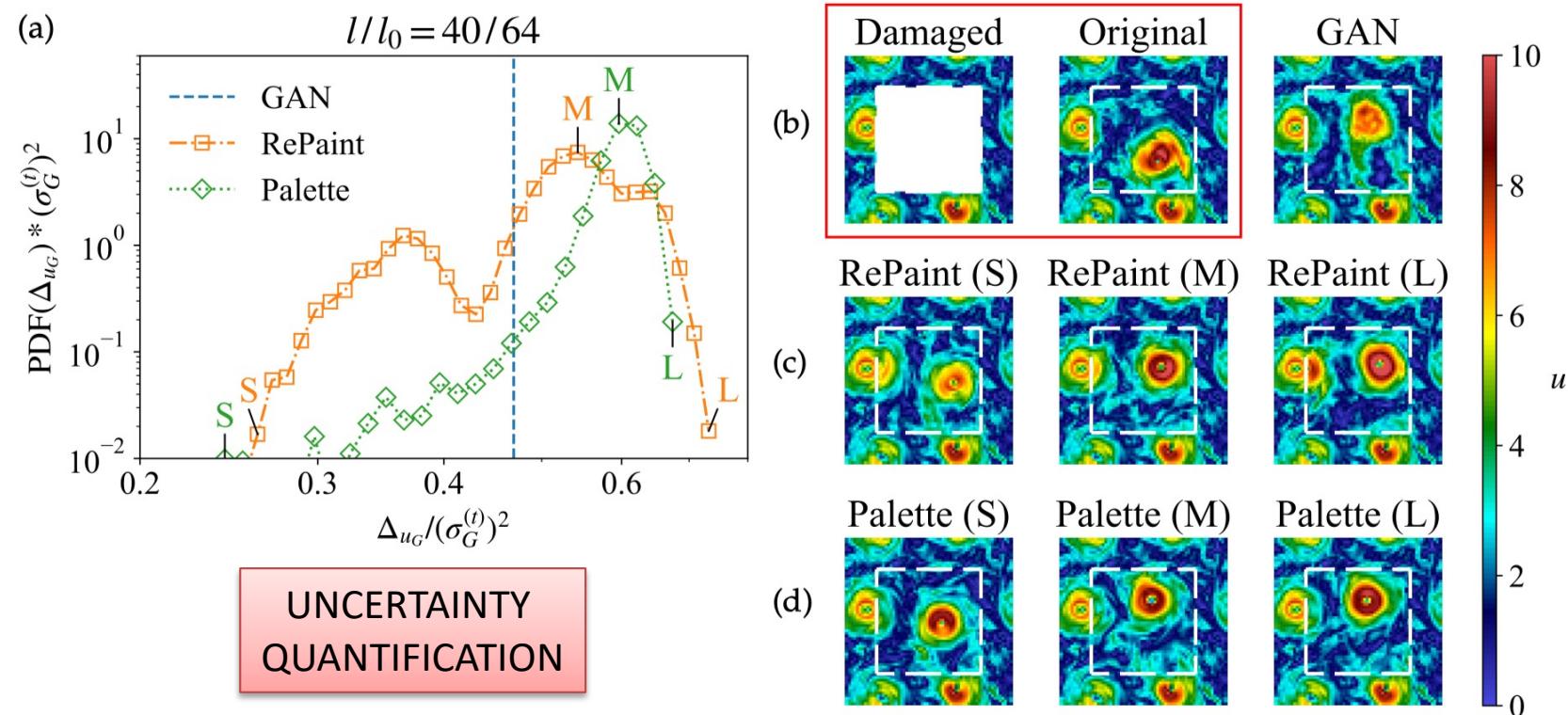
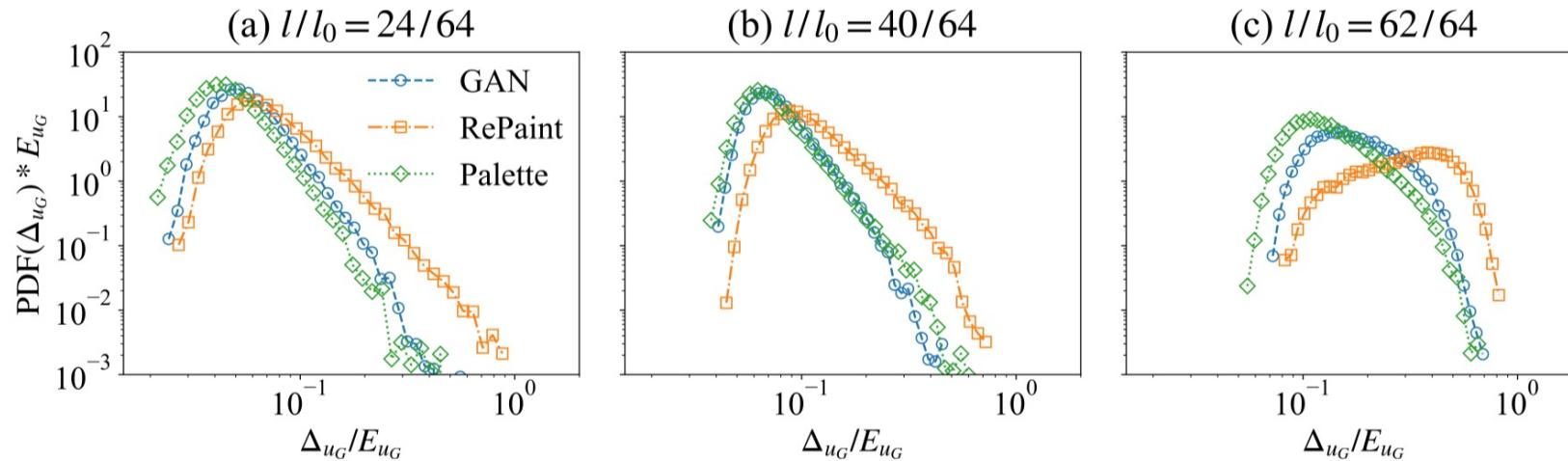


COMPARISON GAN/DM

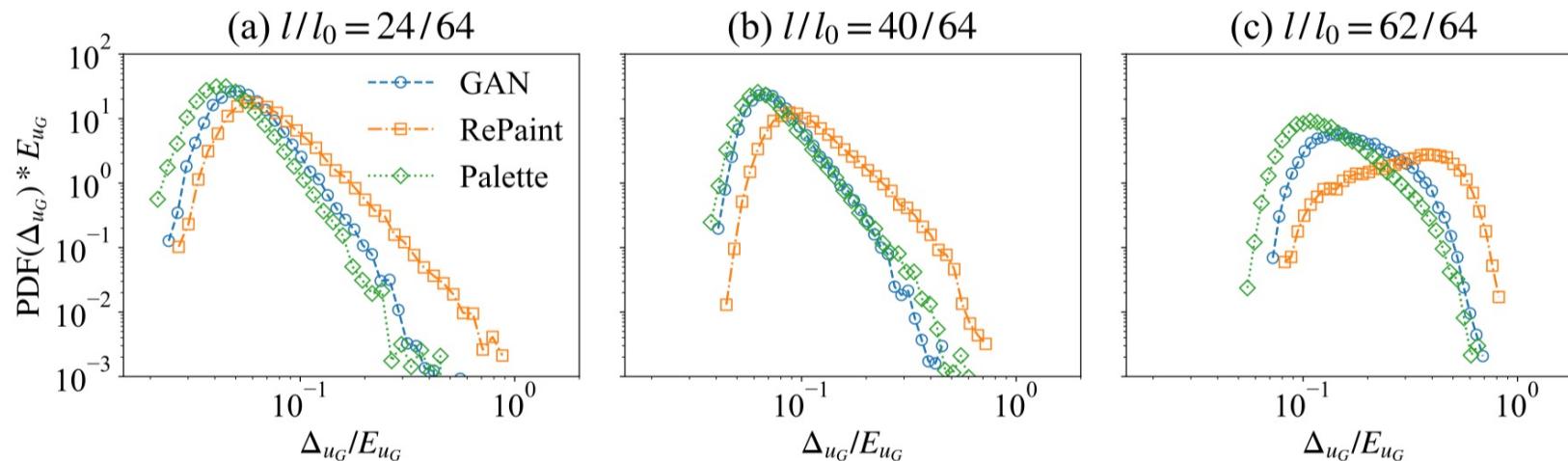
PDF OF L2 ERROR – SINGLE IMAGE



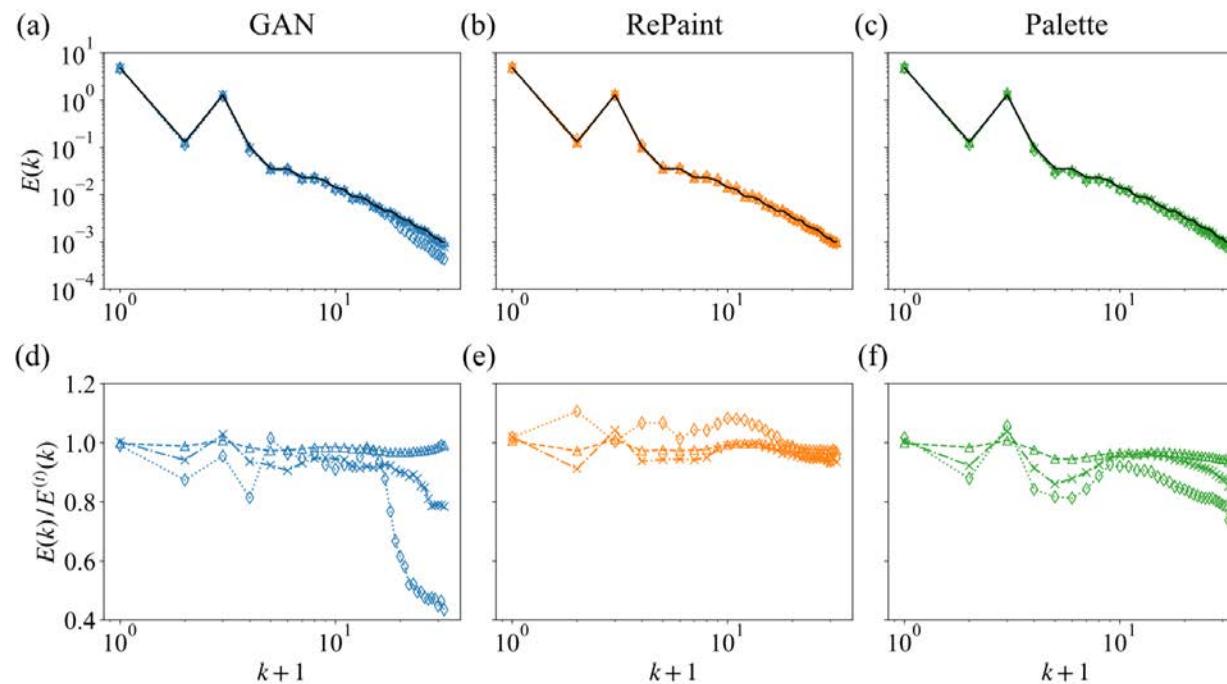
PDF OF L2 ERROR – SINGLE IMAGE



PDF OF L2 ERROR – SINGLE IMAGE

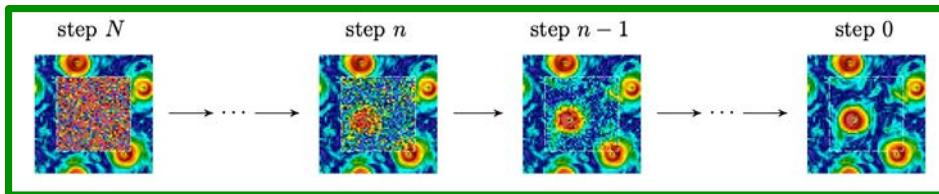
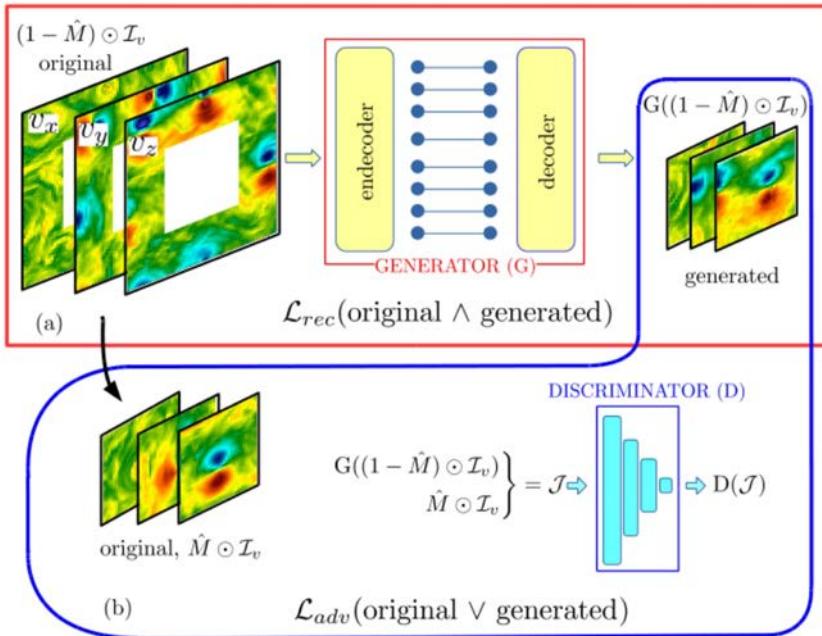


COMPARISON GAN/DM



STATISTICAL RECONSTRUCTION ENERGY SPECTRA

GAN/DM



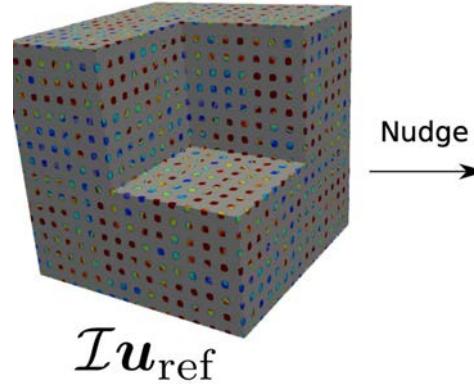
GAN/DM

- EQUATION-FREE
- DATA HUNGRY
- MISSING PHYSICS (SEE LATER)
- +ONCE TRAINED -> INSTANTANEOUS
- +MIXED INPUT FEATURES
- +OK STATISTICAL UNCERTAINTIES

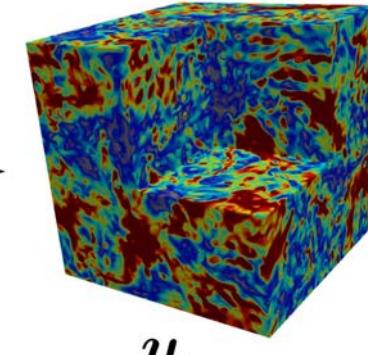
NUDGING

$$\begin{cases} \partial_t \mathbf{v} + \mathbf{v} \cdot \partial_{\mathbf{x}} \mathbf{v} + \partial_{\mathbf{x}} P - \nu \Delta \mathbf{v} = 2\mathbf{v} \times \boldsymbol{\Omega} + \mathcal{S}\mathbf{v} + \alpha g \hat{\mathbf{z}} T + \mathcal{F} - N(\mathbf{v}_N - \mathbf{v}) \\ \partial_t T + \mathbf{v} \cdot \partial_{\mathbf{x}} T - \chi \Delta T = \mathcal{G} v_z + \mathcal{L} - N_T(T_N - T) \end{cases}$$

Nudging field

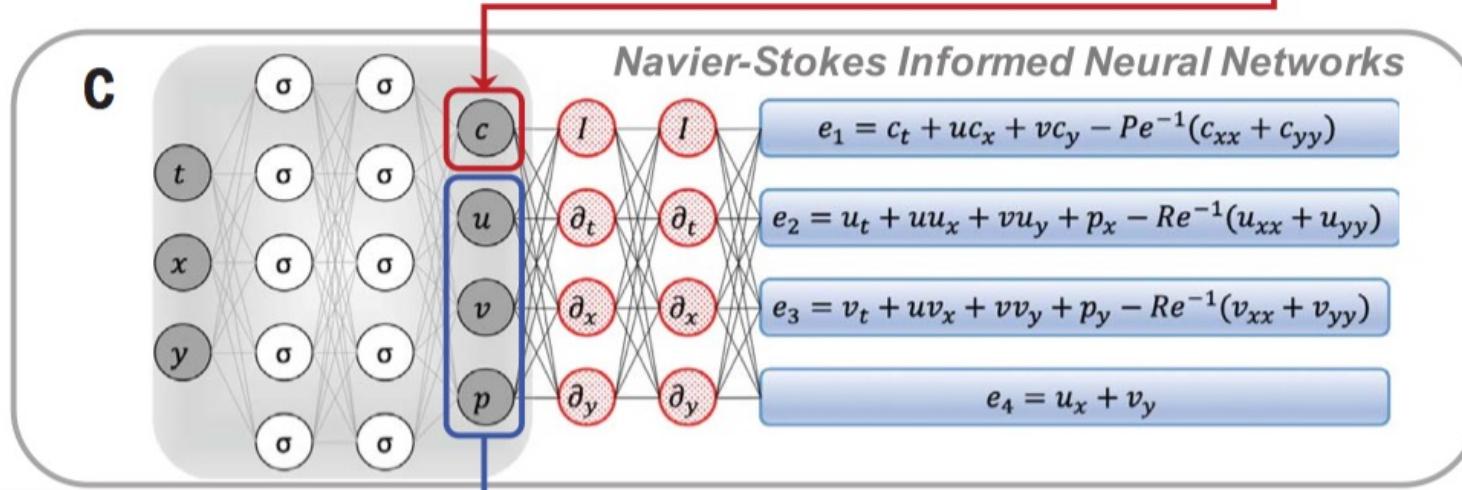
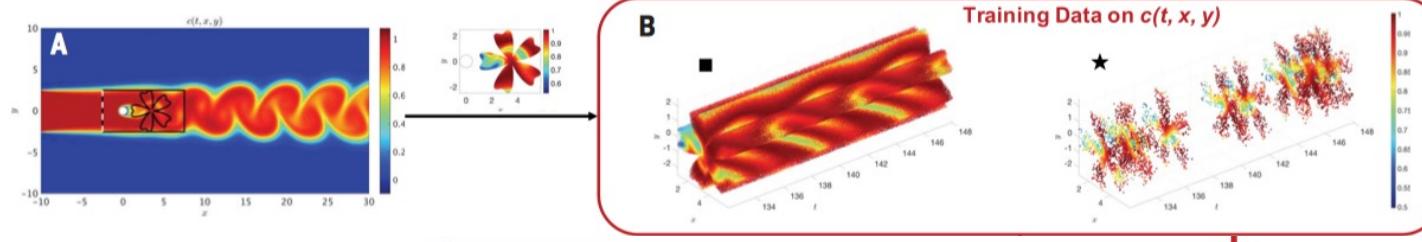
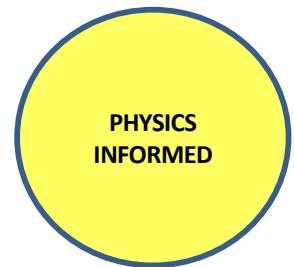


Nudged simulation



NUDGING

- NEEDS EQUATIONS
- CPU HUNGRY
- RESTRICTED INPUT FEATURES
- +PHYSICS COMPLIANT
- +NO TRAINING
- +OK STATISTICAL UNCERTAINTIES



$$MSE = \frac{1}{N} \sum_{n=1}^N |c(t^n, x^n, y^n, z^n) - c^n|^2 + \sum_{i=1}^5 \frac{1}{M} \sum_{m=1}^M |e_i(t^m, x^m, y^m, z^m)|^2$$

ML-TRAINED ON A SPARSE SPATIO+TEMPORAL DATASET
FOR CONCENTRATION \rightarrow INFER VELOCITY + PRESSURE
 \rightarrow BACK PROPAGATE FOR GRADIENTS (**AUTOMATIC DIFFERENTIATION**) \rightarrow NAVIER-STOKES

WHAT WE HAVE:

- + “QUICK” STOCHASTIC TOOL TO GENERATE/AUGMENT REALISTIC 3D EULERIAN CONFIGURATIONS AND 3D LAGRANGIAN TRAJECTORIES IN TURBULENCE, EASY TO GENERALISE FOR DIFFERENT APPLICATIONS
- + HIGH QUANTITATIVE AGREEMENT WITH MULTI-SCALE STATISTICAL PROPERTIES

WHAT WE MISS:

- UNDERSTANDING OF ROBUSTNESS IN GENERALISING OUT-OF-SAMPLE:
EXTREME EVENTS, DIFFERENT REYNOLDS NUMBERS & DIFFERENT FLUIDS PROPERTIES
- UNDERSTANDING SCALING PROPERTIES FOR TIME-TO-SOLUTION AT CHANGING IN-SAMPLE PROPERTIES, I.E. AT CHANGING DIMENSION OF THE TRAINING DATASET,
SETS OF HYPER-PARAMETERS, CNN ARCHITECTURES: GAN, DM, TRANSFORMERS
- WHAT-IF QUESTIONS: EXPLICABILITY OF THE GENERATED DATA, FEATURES RANKINGS,
PHYSICS DISCOVERY



Wavelet Score-Based Generative Modeling

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[arxive 2208.05003](https://arxiv.org/abs/2208.05003)



Guide for users

TURB-Rot. A LARGE DATABASE OF 3D AND 2D SNAPSHOTTS FROM TURBULENT ROTATING FLOWS

A PREPRINT

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Search for datasets

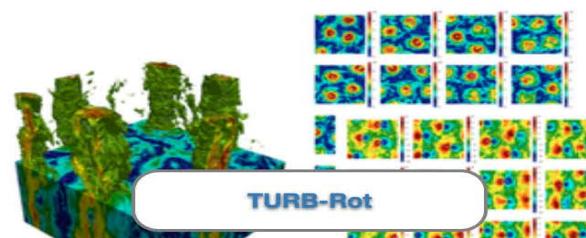


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Datasets

TURB-Rot

A large database of 3d and 2d snapshots from turbulent rotating



2

Organizations

web_admin

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